LIMITI DI FUNZIONI REALI 4

OPERAZIONI SUI LIMITI

$\lim_{x \to x_0} f(x) = l$ $\lim_{x \to x_0} g(x) = m$

$lim┬(x \to x\_0) [f(x) \pm g(x)]$=l$\mp m \to \lim_{x \to x_0} f(x) \pm \lim_{x \to x_0} g(x)$

$\lim\limits_{x \to x_0} f(x)g(x) = lm \to \lim\limits_{x \to x_0} f(x) \lim\limits_{x \to x_0} g(x)$

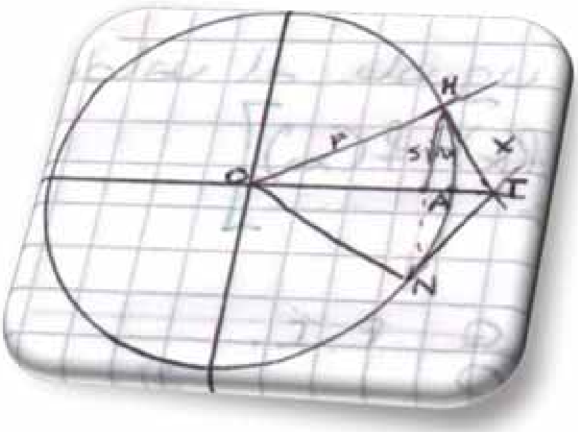
$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \frac{l}{m} \to \frac{\lim_{x \to x_0} f(x)}{\lim_{x \to x_0} g(x)}$

$\lim\limits_{x \to x_0} f(x)$	$\lim\limits_{x \to x_0} g(x)$	$\lim\limits_{x \to x_0} [f(x) + g(x)]$	$\lim\limits_{x \to x_0} [f(x) - g(x)]$	$\lim\limits_{x \to x_0} [f(x)g(x)]$	$\lim\limits_{x \to x_0} \frac{1}{g(x)}$	$\lim\limits_{x \to x_0} \frac{f(x)}{g(x)}$
L	m	l+m	l-m	lm	$\frac{1}{m}$	$\frac{l}{m}$
L	∞	∞	-∞	∞	0	0
∞	m	∞	∞	∞	$\frac{1}{m}$	$\frac{\infty}{0}=\infty$
0	∞	∞	∞	0∞=F.I.	0	$\frac{0}{\infty} = 0 \frac{1}{\infty} = 0$
0	0	0	0	0	∞	$\mathbf{\frac{0}{0}}$**=F.I.**
∞	0	∞	∞	∞0=F.I.	∞	∞
+∞	+∞	+∞	(+∞)-(+∞)=F.I.	+∞	0	$\frac{+\infty}{+\infty} = F.I.$
-∞	-∞	-∞	-∞+∞=F.I.	+∞	0	$\frac{-\infty}{-\infty} = F.I.$
+∞	-∞	+∞-∞=F.I.	+∞+∞=+∞	-∞	0	$\frac{+\infty}{-\infty} = F.I.$
L	0	L	L	0	∞	∞

- $\lim_{X \to X_0} f(x) = k$ f(x)=k
- $\lim_{x \to x_0} x = x_0$
- $\lim_{x \to 0} \sin x = 0$
- $\lim_{x \to 0} \cos x = 1$

LIMITI NOTEVOLI

1. $lim_{x \to 0} \frac{\sin x}{x} = 1$



I= punto di incontro di tangenti

$\frac{\overline{MN}}{r} < \frac{\widehat{MN}}{r} < \frac{\overline{MT}}{r} < \frac{\overline{NT}}{r}$

2sinx$< 2x < 2tgx$

$\frac{sinx}{sinx} < \frac{x}{sinx} < \frac{tgx}{sinx}$

1$< \frac{x}{sinx} < \frac{1}{cosx}$

1$> \frac{sinx}{x} > cosx$

f g h

2. $\boldsymbol{lim_{x\to 0}\frac{1-cos\,x}{x}=0}$

$$\lim_{x\to 0}\frac{(1-\cos x)(1+\cos x)}{x(1+\cos x)}=\lim_{x\to 0}\frac{(1-cos^2 x)}{x(1+\cos x)}=\lim_{x\to 0}\left(\frac{sinx}{x}\right)\left(\frac{sinx}{(1+cosx)}\right)=0$$

3. $\boldsymbol{lim_{x\to 0}\frac{1-cos\,x}{x^2}=\frac{1}{2}}$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}\frac{1+\cos x}{1+\cos x}=\lim_{x\to 0}\frac{1-cos^2 x}{x^2(1+\cos x)}=\lim_{x\to 0}\frac{\sin x}{x}\frac{\sin x}{x}\frac{1}{(1+\cos x)}=\frac{1}{2}$$

4. $\boldsymbol{lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=\mathrm{e}\to 2,718281828485\ldots}$

5. $\boldsymbol{\lim_{x\to 0}\frac{tgx}{x}=1}$

6. $\boldsymbol{lim_{x\to 0}\frac{\sin x+x}{x}=2}$

7. $\boldsymbol{lim_{X\to 0}\frac{\sin x}{x}+\frac{x}{x}=1+1=2}$

8. $\boldsymbol{lim_{x\to 0}\frac{\sin 2x}{tgx}=2}$

9. $\boldsymbol{lim_{x\to 0}\frac{\sin 2x}{2x}\frac{2x}{tgx}=2}$

ESERCIZI

1. $lim_{x\to 0}\left(1+\frac{x}{2}\right)^{1/x}=\sqrt{e}$

$\frac{x}{2}=\frac{1}{y}$

$\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{x}=e$

$$\frac{y}{2}=\frac{1}{x}$$

$$\lim_{x\to\infty}\left[\left(1+\frac{1}{y}\right)^{y}\frac{y}{2}\right]^{\frac{1}{2}}=e^{1/2}=\sqrt{e}$$

2. $lim_{x\to 1}\left[\left(\frac{1}{x-1}\right)-\left(\frac{1}{1-x^2}\right)\right]=\infty$

$$\lim_{x\to 1}\left[\left(\frac{1}{x-1}\right)-\left(\frac{1}{1-x^2}\right)\right]=\mathrm{F.I.}$$

$$\left[\left(\frac{1}{x-1}\right)+\left(\frac{1}{(x+1)(x-1)}\right)\right]$$

$$\left[\frac{(x+1)+1}{(x+1)(x-1)}\right]$$

$$\left[\frac{x+2}{(x+1)(x-1)}\right]=\frac{3}{0}=\infty$$