

LIMITI DI FUNZIONI REALI 2

ESERCIZI SULLA DEFINIZIONE DI LIMITE

Usando la definizione di limite, verificare che :

es 1) $\lim_{x \rightarrow 2} (3x + 1) = 7$

es 2) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$

es 3) $\lim_{x \rightarrow 1} \frac{x^3 - 3x^2 - x + 3}{x^2 - 1} = -2$

es 4) $\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{x} - 1} = 2$

Es. n.1

$$\lim_{x \rightarrow 2} (3x + 1) = 7$$

$$\forall \epsilon > 0, \exists \delta > 0 \mid \forall x \in]2 - \delta; 2 + \delta[- \{2\} \Rightarrow |3x + 1 - 7| < \epsilon$$

$$|3x - 6| < \epsilon$$

$$3|x - 2| < \epsilon$$

$$|x - 2| < \frac{\epsilon}{3}$$

$$|x - 2| = \begin{cases} +(x - 2) & \text{se } x - 2 \geq 0 \\ -(x - 2) & \text{se } x - 2 < 0 \end{cases}$$

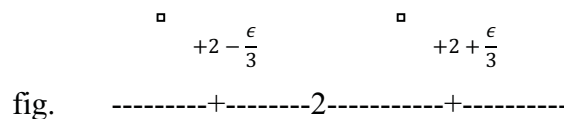
$$|x - 2| = f(x) = \begin{cases} x > 2 \\ x < 2 \end{cases}$$

Se $x > 2$

$$+(x - 2) < \frac{\epsilon}{3}; x < 2 + \frac{\epsilon}{3}$$

Se $x < 2$

$$-(x - 2) < \frac{\epsilon}{3}; x > 2 - \frac{\epsilon}{3}$$



Es..2

$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$$

Scomposizione

$$\lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{x-2} = 4 \quad x - 2 \neq 0 \quad x \neq 2$$

$$\forall x \in \mathbb{R} - \{+2\}$$

$$\forall \epsilon > 0 \exists \delta > 0 | \forall x \in]x_0 - \delta; x_0 + \delta[- \{x_0\} \Rightarrow |f(x) - \ell| < \epsilon$$

$$\forall \epsilon > 0 \exists \delta > 0 | \forall x \in]2 - \delta; 2 + \delta[- \{2\} \Rightarrow \left| \frac{x^2 - 4}{x - 2} - 4 \right| < \epsilon$$

$$\left| \frac{x^2 - 4}{x - 2} - 4 \right| < \epsilon$$

$$|x - 2| = \begin{cases} +(x - 2) & \text{se } x - 2 > 0 \quad (A) \\ -(x - 2) & \text{se } x - 2 < 0 \quad (B) \end{cases}$$

(A)

(B)

$$x > 2$$

$$x < 2$$

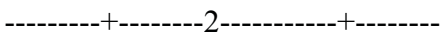
$$(x - 2) < \epsilon$$

$$-(x - 2) < \epsilon$$

$$x < +2 + \epsilon$$

$$x - 2 > -\epsilon$$

$$x > 2 - \epsilon$$

fig. 

Es. n.3

$$\lim_{x \rightarrow 1} \frac{x^3 - 3x^2 - x + 3}{x^2 - 1} = -2 \quad \left| \frac{x^3 - 3x^2 - x + 3}{x^2 - 1} \right| < \epsilon \quad \text{non definita in } x=1$$

$$\left| \frac{x^3 - 3x^2 - x + 3 + 2x^2 - 2}{x^2 - 1} \right| < \epsilon \quad \left| \frac{x^3 - 3x^2 - x + 3}{x^2 - 1} \right| < \epsilon$$

$$(x^2 - 1)(x - 1)$$

$$|x - 1| < \epsilon \quad \begin{cases} x - 1 < \epsilon \\ -x + 1 < \epsilon \end{cases} \quad \begin{matrix} x < 1 + \epsilon \\ x > 1 - \epsilon \end{matrix}$$

$$1 - \epsilon < x < 1 + \epsilon \quad \text{intorno di } 1$$

Es. n.4

$$\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1} = 2$$

$$\left| \frac{x-1}{\sqrt{x}-1} - 2 \right| < \epsilon \qquad \left| (x-1) - 2\sqrt{x} + 2 \right| < \epsilon$$

$$\left| x + 1 - 2\sqrt{x} \right| < \epsilon$$

$$f(x) = \frac{(x-1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = \frac{x\sqrt{x} + x - \sqrt{x} - 1}{(x-1)} = f(x) = 1 + \sqrt{x}$$

$$\left| 1 + \sqrt{x} - 2 \right| < \epsilon$$

$$\begin{cases} \sqrt{x} - 1 < \epsilon \\ -\sqrt{x} + 1 < \epsilon \end{cases} \quad \begin{cases} \sqrt{x} < \epsilon + 1 \\ -\sqrt{x} < \epsilon - 1 \end{cases} \quad \begin{cases} x < (1 + \epsilon)^2 \\ x > (1 - \epsilon)^2 \end{cases} \quad \text{evidente intervallo di } 1$$