LIMITI DI FUNZIONI REALI 2

ESERCIZI SULLA DEFINIZIONE DI LIMITE

Usando la definizione di limite, verificare che :

es 1) $\lim_{x\to 2}(3x+1)=7$

es 2) $\lim_{x\to 2}\frac{x^2-4}{x-2}=4$

es 3) $\lim_{x\to 1}\frac{x^3-3x^2-x+3}{x^2-1}=-2$

es 4) $\lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}=2$

Es. n.1

$\lim_{x\to 2}(3x+1)=7$

$\forall\epsilon>0, \exists\delta>0 \mid \forall\, x\,\epsilon\,]2-\delta;2+\delta[-\{2\} \Rightarrow |3x+1-7|<\epsilon$

$|3x-6|<\epsilon$

$3|x-2|<\epsilon$

$|x-2|<\frac{\epsilon}{3}$

$|x-2|=\begin{cases}+(x-2)\ se\ x-2\geq 0\\ -(x-2)\ se\ x-2<0\end{cases}$

$|x-2|=f(x)=\begin{cases}x>2\\ x<2\end{cases}$

Se $x>2$

$+(x-2)<\frac{\epsilon}{3}$; $x<+2+\frac{\epsilon}{3}$

fig.

$+2-\frac{\epsilon}{3}$ $+2+\frac{\epsilon}{3}$

---------+--------2-----------+----------

Se $x<2$

$-(x-2)<\frac{\epsilon}{3}$; $x>2-\frac{\epsilon}{3}$

Es..2

$$\lim_{x\to 2}\frac{x^2-4}{x-2}=4$$

Scomposizione

$\lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}=4 \qquad x-2\neq 0 \qquad x\neq 2$

$\forall x \epsilon \mathbb{R}-\{+2\}$

$\forall\epsilon>0\exists\delta>0|\forall x\epsilon]x_0-\delta;x_0+\delta[-\{x_0\}\Rightarrow|f(x)-\ell|<\epsilon$

$\forall\epsilon>o\exists\delta>o|\forall x\epsilon]2-\delta;2+\delta[-\{2\}\Rightarrow\left|\frac{x^2-4}{x-2}-4\right|<\epsilon$

$$\left|\frac{x^2-4}{x-2}-4\right|<\epsilon$$

$$|x-2|=\begin{cases}+(x-2)\ se\ x-2>0 & (A)\\ -(x-2)\ se\ x-2<0 & (B)\end{cases}$$

(A)	(B)
$x>2$	$x<2$
$(x-2)<\ \epsilon$	$-(x-2)<\ \epsilon$
$x<+2+\epsilon$	$x-2>-\epsilon$
	$x>2-\epsilon$

$+2-\epsilon$ $+2+\epsilon$

fig. ---------+--------2-----------+--------

Es. n.3

$\lim_{x\to 1}\frac{x^3-3x^2-x+3}{x^2-1}=-2 \qquad \left|\frac{x^3-3x^2-x+3}{x^2-1}\right|<\ \epsilon$ non definita in x=1

$\left|\frac{x^3-3x^2-x+3+2x^2-2}{x^2-1}\right|<\ \epsilon \qquad \left|\frac{x^3-3x^2-x+3}{x^2-1}\right|<\ \epsilon$

$(x^2-1)(x-1)$

$|x-1|<\ \epsilon \qquad \begin{cases}x-1<\epsilon\\ -x+1<\epsilon\end{cases} \qquad \begin{matrix}x<1+\epsilon\\ x>1-\epsilon\end{matrix}$

$1-\epsilon<x<1+\epsilon$ intorno di 1

Es. n.4

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1} = 2$$

$\left|\frac{x-1x}{\sqrt{x}-1} - 2\right| < \epsilon$ $\qquad\qquad$ $\left|(x-1) - 2\sqrt{x} + 2\right| < \epsilon$

$$\left|x + 1 - 2\sqrt{x}\right| < \epsilon$$

$$f(x) = \frac{(x-1)\left(\sqrt{x}+1\right)}{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)} = \frac{x\sqrt{x} + x - \sqrt{x} - 1}{(x-1)} = f(x) = 1 + \sqrt{x}$$

$$\left|1 + \sqrt{x} - 2\right| < \epsilon$$

$\begin{cases} \sqrt{x} - 1 < \epsilon \\ -\sqrt{x} + 1 < \epsilon \end{cases}$ $\begin{cases} \sqrt{x} < \epsilon + 1 \\ -\sqrt{x} < \epsilon - 1 \end{cases}$ $\begin{cases} x < (1+\epsilon)^2 \\ x > (1-\epsilon)^2 \end{cases}$ evidente intervallo di 1